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A low-cycle fatigue approach to fatigue crack propagation

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The damage accumulation hypothesis is used to derive a fatigue crack growth rate equation. The fatigue life of a volume element inside the plastic zone is evaluated by using low-cycle fatigue concepts. Crack growth rate is expressed as a function of cyclic material parameters and plastic zone characteristics. For a given material, crack growth increment is predicted to be a fraction of the plastic zone size which can be expressed in terms of fracture mechanics parameters, K and J. Hence, the proposed growth rate equation has a predictive capacity and is not limited to linear elastic conditions.

1. Introduction

Fatigue crack propagation (FCP) has been of interest to researchers from various disciplines, namely material scientists, mechanical engineers, and fracture mechanics specialists. Material scientists have been primarily concerned with the mechanisms of FCP and have placed the emphasis on microstructure, while others have attempted to develop models which could predict the FCP lives of cracked bodies using continuum concepts.

It has been a usual practice to correlate crack growth rate with some function of the stress intensity factor range, ΔK, ever since Paris and Erdogan [1] proposed a growth rate equation of the following form

$$\mathrm{d}l/\mathrm{d}N = C(\Delta K)^m \tag{1}$$

where l is the crack length and N the number of cycles. C and m are regarded as material constants and the latter is a more important parameter because it indicates the stress range dependence of the growth rate. Equation 1, known as the Paris (or Paris–Erdogan) law, has enjoyed a tremendous popularity due to its simplicity and wide-range applicability. It has had a significant impact on FCP research and the theories that followed were almost invariably addressed to its derivation using different physical criteria for crack extension.

Several FCP theories are based on the geometrical consequences of the crack tip deformation to predict crack growth rates. McClintock [2] postulated that the crack growth increment should be related to the crack tip opening displacement (CTOD) and reported correlation between striation spacing and CTOD data. A relation between crack growth increment and CTOD was suggested and derived by other investigators [3–11] who have used the plastic blunting process [12] of FCP or its versions in their analyses. It was initially indicated that the crack advance per cycle should be approximately one-half of the CTOD with all the plastic deformation occurring at the crack tip. However, the crack increment was experimentally shown [13] to be a very small fraction of the CTOD in the linear elastic range. Apparently, the contribution of the crack tip plastic deformation to crack growth is small compared to the CTOD it creates under linear elastic conditions. Kuo and Liu [8] argued that only a fraction of the crack tip plastic deformation results in crack propagation. According to their model, the slip activities behind the crack tip contribute to crack opening but do not cause crack growth.

A large number of FCP theories are not based on a specific mechanism in contrast to the COD theories. Among these are "energy balance" and "damage accumulation" models. Energy balance hypothesis has its origin in the classical work of Griffith [14] which is applicable only to perfectly brittle materials. FCP in metals is often accompanied by plastic deformation and energy is dissipated in deformation processes in addition to new surface creation. In fact, the latter can be neglected in ductile materials. The energy input must be greater than or equal to the energy dissipated in the form of heat together with the formation energies of the crack tip plastic zone and new crack surface for a fatigue crack to extend. This concept, in one form or another, has been used [15–19] in the derivation of crack growth rate equations.

The "damage accumulation" concept has also been very popular in FCP studies. Because stresses, which are safe when applied only once, result in failures upon repeated application, it is tempting to think that some sort of damage accumulation takes place in each stress cycle. It is assumed in damage accumulation theories that fracture occurs when the cumulative damage at the crack tip, at any stage of the propagation, reaches a critical value. With a damage parameter, D, a common crack extension criterion can be formulated as

$$\sum_{n} D = D_c \tag{2}$$

where n is the number of fatigue cycles, D is the

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damage increment per cycle and D_c is the critical value of the damage parameter. Plastic strain [20–27], displacement [28–31], and plastic work [32–35] terms have been selected as the damage parameter in various analyses.

A group of damage accumulation theories have incorporated low-cycle fatigue (LCF) concepts into modelling of FCP. Liu *et al.* [21, 22] combined the Coffin [36]–Manson [37] law and Miner's rule [38] and considered a volume element which is deformed with increasing strain amplitudes while traversing the plastic zone. Their results indicate that FCP resistance of a material is related to its cyclic ductility. Many versions [23–26] of their model have been introduced, predicting that crack growth rate is controlled by cyclic and tensile properties. Chalant and Remy [39] recently incorporated metallurgical considerations into the LCF approach to FCP. They concluded that the validity of these models is confined to the low crack growth rates and a crack grows with plastic stretching mechanisms at high growth rates.

In the present analysis, an attempt is made to derive a crack growth rate equation using LCF approach to FCP. The objectives of this work can be listed as follows:

(1) to provide a plausible and qualitative description of ductility exhaustion model of crack advance;

(2) to express crack growth rate as a function of material parameters and variables which are easy to measure or readily available, while restoring a predictive ability.

2. Modelling of fatigue crack propagation

The model proposed is based on the premise that a volume element along the crack trajectory fractures when its ductility is exhausted. Owing to the nature of crack tip strain distribution, this volume element is more likely to be at the crack tip adding its size to the crack length when it fractures. However, the possibility of microcracking ahead of the crack tip always exists. The ductility along the crack trajectory varies in the case of non-homogeneous media. Microcracks may form at brittle microstructural constituents ahead of the crack tip under damage accumulation conditions. This, however, will be considered as an exception and homogeneous microstructure will be assumed in the present analysis. This assumption is valid because microcracking ahead of the crack tip is not the mechanism with which fatigue cracks propagate in metals, but occurs only occasionally.

It is presupposed in all damage accumulation models that there is a plastic enclave at the crack tip. Without a crack tip plastic zone the process of damage accumulation cannot be accounted for. It is also assumed in the present analysis that:

(1) the crack propagates under a constant driving force. This directly implies that the crack growth increment and plastic zone size remain constant during crack propagation and the crack tip strain field is translated only;

(2) the crack propagates every cycle.

A volume element deforms only elastically away from the crack tip. It starts experiencing plastic strains when it enters the crack tip plastic zone. When it does, it can be viewed as a low-cycle fatigue specimen which is damaged permanently every cycle as the deformation inside the plastic zone is both mechanically and thermodynamically irreversible. The present model evaluates the fatigue history of such an element along the crack trajectory from the moment it crosses the plastic zone boundary until it arrives at the crack tip where it fractures.

Consider an element, "A", which has just entered the plastic zone (Fig. 1a). "A" is approached by the crack tip every cycle and experiences larger and larger plastic strains as it traverses the plastic zone. After $(n - 1)$ fatigue cycles, the crack tip arrives at "A" which fractures the next cycle (Fig. 1e), causing the crack to grow by Δl. n cycles have elapsed from the moment "A" enters the plastic zone until it fractures at the crack tip. In the meantime, the crack tip travels a distance r_p. Hence, crack advance per cycle can be expressed as

$$\Delta l = r_p/n \tag{3}$$

where r_p is the monotonic plastic zone size in the crack growth direction. r_p can be calculated from fracture mechanics or from various empirical relations [40] which have resulted from experimental studies. n represents the fatigue life of "A" which is considered as a low-cycle fatigue specimen. Accordingly, it can be evaluated by employing Miner's rule to account for the accumulation of damage within "A" during fatigue cycling and the Coffin–Manson law as a fracture criterion.

"A" experiences a different and larger plastic strain amplitude (PSA) every cycle. Because "A" has a finite size (Δl), an average PSA will be assigned to its successive positions while traversing the plastic zone with an increasing number of cycles (Fig. 2a). As it spends only one cycle at each PSA, Miner's rule for the present situation can be written as

$$1/N_{F1} + 1/N_{F2} + \ldots + 1/N_{Fn} = \sum_{i=1}^{n} 1/N_{Fi} = 1 \tag{4}$$

N_{Fi} is the number of fatigue cycles it would take "A" to fracture if it were fatigued at a constant PSA, ε_i (Fig. 2b). n, on the other hand, takes into account the accumulation of damage introduced at increasing strain levels under low-cycle fatigue conditions. The value of n is such that Equation 4 is satisfied and is likely to be smaller than $(N_{F1} - N_{Fn})$. However, n can be equal to $(N_{F1} - N_{Fn})$ when the crack growth increment, i.e. size of "A" in the crack growth direction, is infinitesimal. Hence, we can express n as

$$n = \alpha\,(N_{F1} - N_{Fn}) \tag{5}$$

where α is a proportionality constant which is smaller than or equal to 1. α can be evaluated very accurately from the following approximation

$$\alpha \simeq \frac{\sum_{i=1}^{n} 1/N_{Fi}}{\int_{N_{Fn}}^{N_{F1}} (1/N_F)\,dN_F} \tag{6}$$

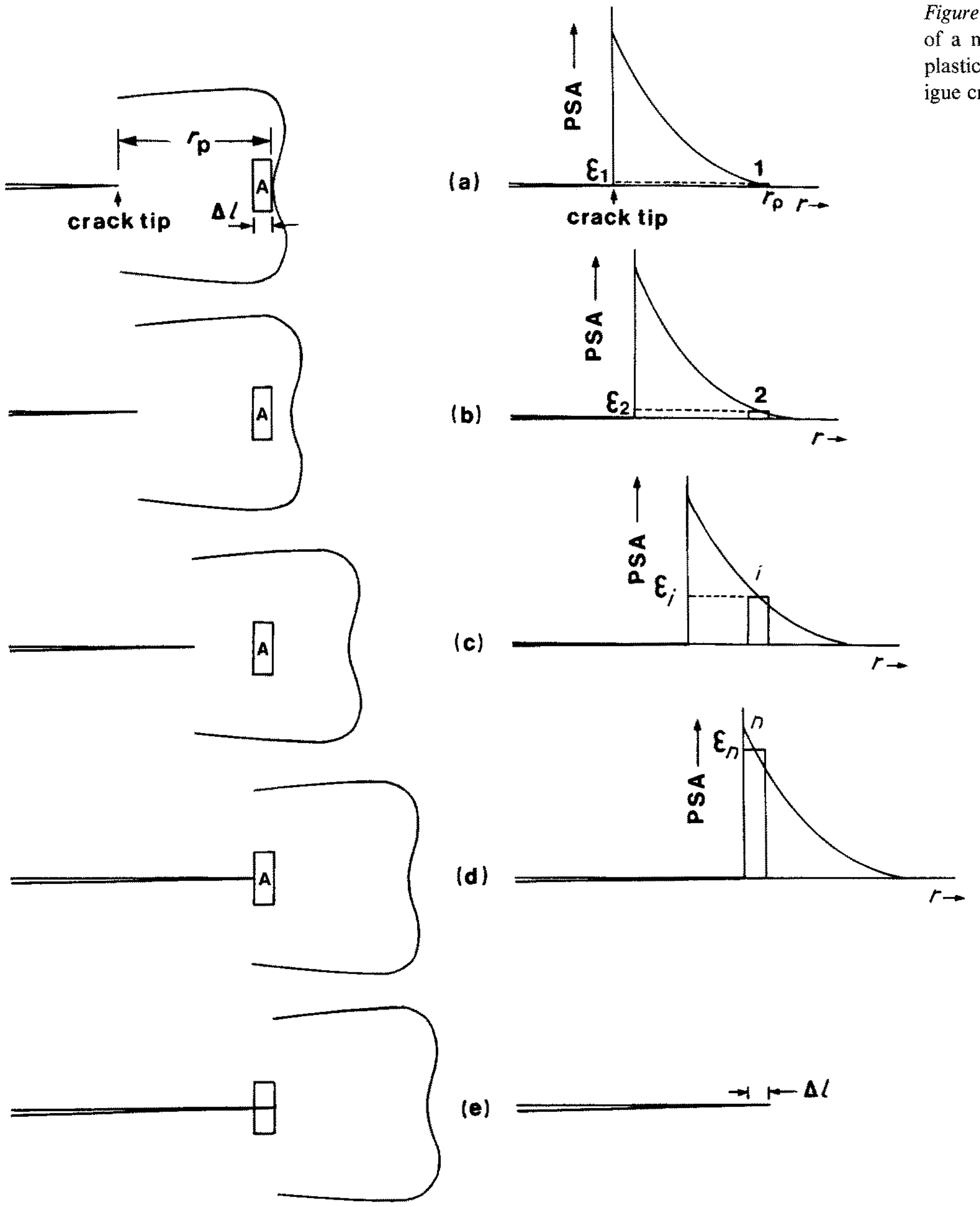

Figure 1 The deformation history of a material element inside the plastic zone of a propagating fatigue crack.

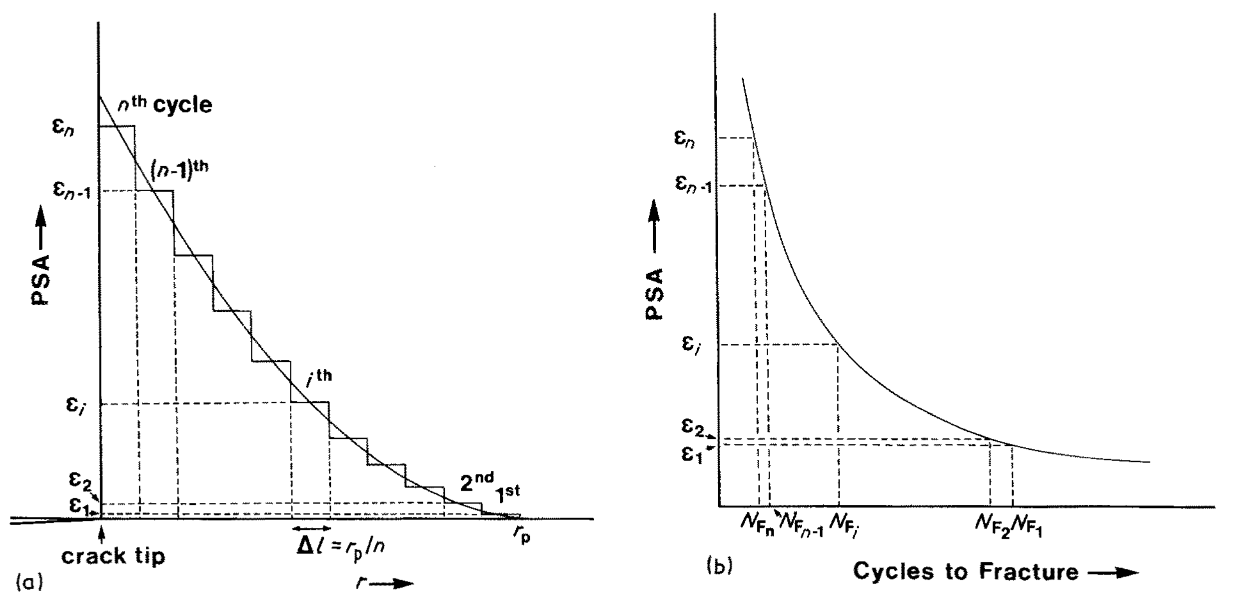

Figure 2 (a) The plastic strain amplitudes (PSA) experienced by a material element as it traverses the plastic zone, and (b) the corresponding low-cycle fatigue lives.

Assuming exact equality, analysis of Equation 6 yields

$$\alpha = \frac{1}{\ln(N_{\mathrm{F1}}/N_{\mathrm{F}n})} \quad (7)$$

By substituting this expression for α in Equation 5, we obtain

$$n = \frac{N_{\mathrm{F1}} - N_{\mathrm{F}n}}{\ln(N_{\mathrm{F1}}/N_{\mathrm{F}n})} \quad (8)$$

N_{F1} and $N_{\mathrm{F}n}$ are associated with the start and the end of low-cycle fatigue life of "A", respectively. They can be expressed in terms of low-cycle fatigue properties by employing the Coffin–Manson law

$$N_{\mathrm{F}i} = (C/\varepsilon_i)^{1/\beta} \quad i = 1, n^* \quad (9)$$

where C and β are fatigue ductility coefficient and exponent, respectively. Combining Equations 3, 8, and 9, we obtain the following expression for crack advance per cycle

$$\Delta l = r_{\mathrm{p}} \frac{\ln(\varepsilon_{\mathrm{F}}/\varepsilon_{\mathrm{PZB}})}{\beta C^{1/\beta}\,[1/(\varepsilon_{\mathrm{PZB}})^{1/\beta} - 1/(\varepsilon_{\mathrm{F}})^{1/\beta}]} \quad (10)$$

ε_{F} and $\varepsilon_{\mathrm{PZB}}$ are the PSAs experienced at the crack tip and the plastic zone boundary, respectively. Equation 10 expresses crack growth rate as a function of plastic zone size (r_{p}), crack tip strain distribution (ε_{F}, $\varepsilon_{\mathrm{PZB}}$), and low-cycle fatigue properties (β, C). It can be rewritten simply as

$$\mathrm{d}l/\mathrm{d}N = f_1(\varepsilon_{\mathrm{F}}, \varepsilon_{\mathrm{PZB}}, \beta, C)\, r_{\mathrm{p}} \quad (11)$$

The crack growth rate is predicted to be independent of the details of the strain distribution inside the plastic zone. It is affected only by the boundary values of this distribution at the crack tip and at the plastic zone boundary. For a given material, the factors that control crack growth rate are obviously all related to the crack tip plasticity, because β and C are constants.

The plastic zone size, r_{p}, is usually correlated with the stress intensity factor, K, assuming that the plastic zone is small compared to the crack length [40]:

$$r_{\mathrm{p}} = \text{constant}\,(K_{\max}/\sigma_{\mathrm{y}})^2 \quad (12)$$

$K_{\max}$ is the maximum stress intensity factor and σ_{y} is the yield strength of the material. When the plastic zone is large enough to be comparable to the crack length, J replaces K as the stress intensity parameter

$$r_{\mathrm{p}} = \text{constant}\,(E/\sigma_{\mathrm{y}}^2)\,J_{\max} \quad (13)$$

E is Young's modulus and $J_{\max}$ is the maximum energy release rate. Incorporating these expressions for the plastic zone size in Equation 11, we obtain

$$\mathrm{d}l/\mathrm{d}N = f_2(\varepsilon_{\mathrm{F}}, \varepsilon_{\mathrm{PZB}}, \sigma_{\mathrm{y}}, \beta, C)\, K_{\max}^2 \quad (14\mathrm{a})$$

$$\mathrm{d}l/\mathrm{d}N = f_3(\varepsilon_{\mathrm{F}}, \varepsilon_{\mathrm{PZB}}, \sigma_{\mathrm{y}}, E, \beta, C)\, J_{\max} \quad (14\mathrm{b})$$

for linear elastic and elastoplastic crack extension processes, respectively. In addition to β and C, σ_{y} and E are material parameters which are also independent of the crack propagation process. Because the crack tip strain field experiences only translation when the driving force for FCP ($K_{\max}$ or $J_{\max}$) is constant; ε_{F} and $\varepsilon_{\mathrm{PZB}}$ are also invariant with respect to FCP. This results in a constant crack growth rate as expected. In real life, however, cracks very seldom propagate under constant driving-force conditions. The intensity of the stress–strain field ahead of the crack tip usually increases with crack propagation, causing the crack to accelerate. The movement of the crack tip plastic zone (CTPZ) cannot be characterized simply in terms of translation alone under such circumstances. The argument that the critical strain at which crack advances (ε_{F}) remains constant throughout the crack propagation life may not be valid when the evolution of the CTPZ involves expansion and distortion in addition to translation. It is well established that the residual stresses at the crack tip are dictated by the CTPZ geometry. Stresses which are opposite in sign to those applied often develop at the crack tip when the plastic enclave, which experiences expansion at least in the loading direction, tries to fit within the elastic surrounding upon unloading. Compressive residual stresses, for example, are commonly encountered in tension–tension fatigue and affect the kinetics of crack propagation. It is well known that the capacity of materials for plastic deformation increases significantly in the presence of compressive stresses. The magnitude of these stresses and its variation with crack propagation is very much a function of the plastic zone evolution. The above discussion suggests that ε_{F} may be increasing with crack propagation unless the crack driving force is maintained constant. On the other hand, the plastic strains in the vicinity of the PZB are not affected by increasing crack length to a significant extent. Increasing stress intensities simply move the PZB further away from the crack tip. Hence, we can state that the change in $\varepsilon_{\mathrm{PZB}}$ with crack propagation, if any, can be neglected. ε_{F} and $\varepsilon_{\mathrm{PZB}}$, can either be obtained experimentally (extrapolation may be necessary for the estimation of ε_{F}!) or calculated from the analysis of plastic flow behaviour of materials [34]. For a given material, it is possible to predict crack growth rates with fracture mechanics parameters by using Equations 14a and b.

3. Discussion

Various shapes have been reported [40, 41] for the crack tip plastic zones. Sometimes, the plastic deformation that accompanies FCP takes place on two sides of the crack, leaving the material directly ahead of the crack tip undeformed (Fig. 3). When it does, the damage accumulation hypothesis cannot be valid because the crack will be propagating through a region which shows no signs of damage or damage accumulation. Other mechanisms such as the plastic blunting process could be responsible for crack extension in those cases. The model introduced here, presupposes that the material in front of the crack tip is deformed. Transmission electron microscopy studies of the crack tip region [42] have shown that the dislocation configurations in this region resemble those of low-cycle fatigue specimens, fatigued to saturation.

*The subscripts 1 and n refer to the conditions at the plastic zone boundary and at the crack tip, respectively. Therefore, 1 will be replaced with PZB, (plastic zone boundary) and n with F (fracture) from here on.

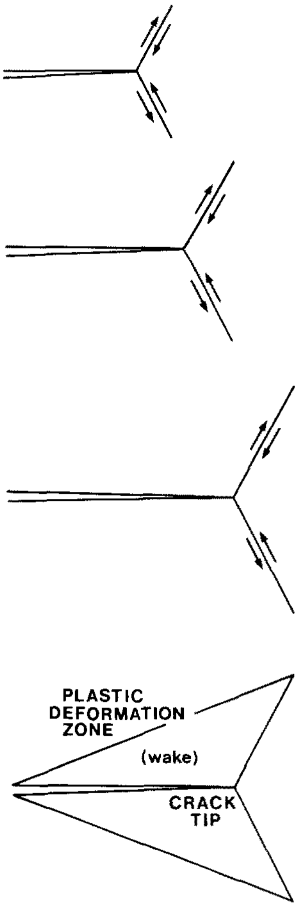

Figure 3 Fatigue crack propagation by crack tip slip activities and the resulting plastic zone configuration.

Crack tip cell structure is typical of high-plastic-strain amplitudes with a gradual change to lower-amplitude configurations away from the crack tip. Hence, the low-cycle fatigue approach to the modelling of FCP is an accurate one when the crack tip is deformed.

In the present analysis, the crack growth increment is predicted to be a fraction of the plastic zone size. Because material elements start experiencing plastic strain amplitudes as soon as they cross the monotonic plastic zone boundary, K_{max} (J_{max}) instead of ΔK (ΔJ) appears in the growth rate equation. However, K_{max} and ΔK are simply related according to

$$K_{max} = \Delta K/(1 - R) \quad (15)$$

where R is the ratio of the minimum to maximum stress intensity factor. So, Equation 14a can be rearranged in the following form

$$dl/dN = f_4(\varepsilon_F, \varepsilon_{PZB}, \sigma_y, \beta, C, R)\, \Delta K^2 \quad (16)$$

to account for the mean stress effects.

A linear relationship between crack growth rate and plastic zone size naturally yields a second-power dependency on the stress intensity factor range. There is a large number of FCP theories that predict a second-power Paris Law. The COD theories [2–11] and the damage accumulation model with critical strain criterion [20–27] are among them. Cumulative displacement and plastic work theories [28–35], on the other hand, result in a fourth-power relationship. Considering the variance of existing experimental results, it is always possible to find a set of FCP data that supports a particular view. Therefore, experimental data correlation was not attempted in the present work.

4. Conclusion

The damage accumulation hypothesis is used to derive a FCP rate equation. Crack growth rate is expressed as a function of cyclic material properties and crack tip plastic zone characteristics. For a given material, crack advance per cycle is predicted to be linearly related to the plastic zone size. The latter can be expressed in terms of stress intensity parameters, K and J. Hence, the rate equation presented is not limited to linear elastic conditions but can also be applied in the elastic–plastic range.

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